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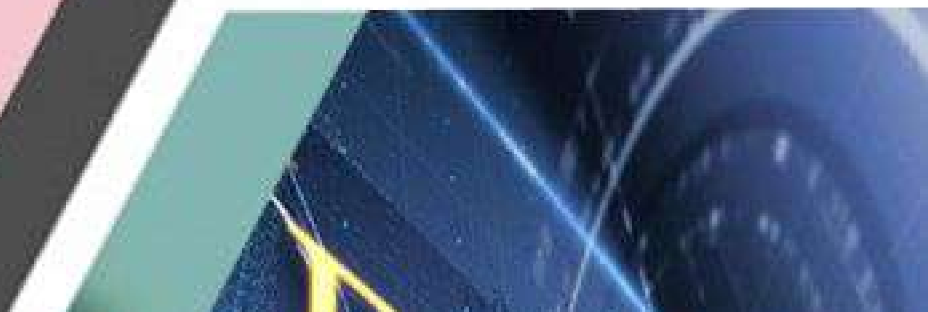
IIT JAM

PHYSICS

SOLVED SAMPLE PAPER



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IIT JAM-PH PHYSICS (FMTP)

Attempt ALL the 60 questions.

There are a total of 60 questions carrying 100 marks.

Section-A contains a total of 30 **Multiple Choice Questions (MCQ).** Q.1 - Q.10 carry 1 mark each and Questi ons Q.11 - Q.30 carry 2 marks each.

Section-B contains a total of 10 **Multiple Select Questions (MSQ).** Questions Q.31 - Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

Section-C contains a total of 20 **Numerical Answer Type (NAT)** questions. Questions Q.41 - Q.60 belong to this section and carry a total of 30 marks. Q.41 - Q.50 carry 1 mark each and Questions Q.51 - Q.60 carry 2 marks each.

In **Section-A** for all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section-B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section-C** (NAT) as well.

Time : 3 Hours **MAX.MARKS : 100** **MARKS SCORED :** ☐

SECTION-A (Q.1-30) MULTIPLE CHOICE QUESTIONs (MCQs) :

1. The entropy of a system, S, is related to the accessible phase space volume G by $S = k_B \ln G(E, N, V)$ where E, N and V are the energy, number of particles and volume respectively. From this one can conclude that G

(A) does not change during evolution to equilibrium

(B) oscillates during evolution to equilibrium

(C) is a maximum at equilibrium

(D) is a minimum at equilibrium

2. Let dw be the work done in a quasistatic reversible thermodynamic process. Which of the following statements about dw is correct?

(A) dw is a perfect differential if the process is isothermal.

(B) dw is a perfect differential if the process is adiabatic.

(C) dw is always a perfect differential.

(D) dw cannot be a perfect differential.

3. A vector perpendicular to any vector that lies on the plane defined by $x + y + z = 5$, is

(A) $\hat{i}+\hat{j}$ (B) $\hat{j}+\hat{k}$ (C) $\hat{i}+\hat{j}+\hat{k}$ (D) $2\hat{i}+3\hat{j}+5\hat{k}$

4. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ are

(A) (1, 4, 9) (B) (0, 7, 7) (C) (0, 1, 3) (D) (0, 0, 14)

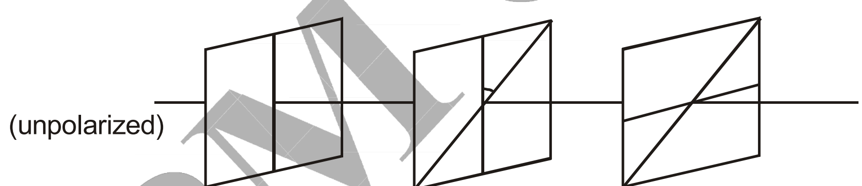
5. Two events, separated by a (spatial) distance 9×10^9 m, are simultaneous in one inertial frame. The time interval between these two events in a frame moving with a constant speed 0.8 c (where the speed of light $c = 3 \times 10^8$ m/s) is

(A) 60 s (B) 40 s (C) 20 s (D) 0 s

6. The commutator $[x^2, p^2]$ is

(A) $2i\hbar[x,P]$ (B) $2i\hbar(xp + px)$ (C) $2i\hbar$ (D) $2i\hbar(xp - px)$

7. Consider three polarizers P_1, P_2 and P_3 placed along an axis as shown in the figure



The pass axis of P_1 and P_3 are at right angles to each other while the pass axis of P_2 makes an angle θ with that of P_1. A beam of unpolarized light of intensity I_0 is incident on P_1 as shown. The intensity of light emerging from P_3 is

(A) 0 (B) $\frac{I_0}{2}$ (C) $\frac{I_0}{8}\sin^2 2\theta$ (D) $\frac{I_0}{4}\sin^2 2\theta$

8. A point particle of mass m carrying an electric charge q is attached to a spring of stiffness constant k. A constant electric field E along the direction of the spring is switched on for a time interval $T(\text{where } T << \sqrt{m/k})$. Neglecting radiation loss, the amplitude of oscillation after the field is switched off is

(A) $\frac{qE}{k}$ (B) $\frac{qET^2}{m}$ (C) $\frac{qE\sqrt{m}}{Tk^2}$ (D) $\frac{qET}{\sqrt{mk}}$

9. A transistor with $\beta = 100$ is used in CE configuration. The collector circuit resistance is $1k\Omega$ & $V_{cc} = 20v$. Assuming $V_{be} = 0$ find the value of collector to base resistance such that quiescent collector to emitter voltage is 4v

(A) $25K\Omega$ (B) $16k\Omega$ (C) $10k\Omega$ (D) $5k\Omega$

10. The phenomenon known as 'Early effect' in a bipolar transistor refers to a reduction of the effective-basewidth caused by___

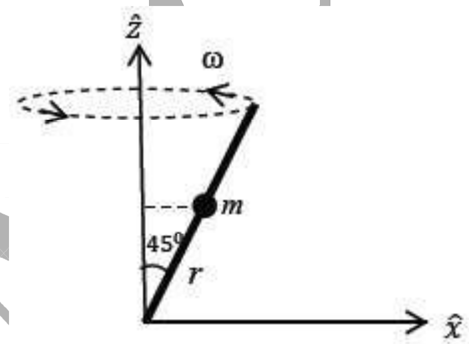
(A) Electron-hole recombination at the base

(B) The reverse biasing of the base-collector junction

(C) The forward biasing of emitter-base junction

(D) The early removal of stored base charge during saturation to cut-off switching

11. A bead of mass m can slide without friction along a massless rod kept at 45^0 with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed ω. At any instant, r is the distance of the bead from the origin. The momentum conjugate to r is



(A) $m\dot{r}$ (B) $\frac{1}{\sqrt{2}}m\dot{r}$ (C) $\frac{1}{2}m\dot{r}$ (D) $\sqrt{2}m\dot{r}$

12. A particle of mass m is in a potential given by

$$V(r) = -\frac{a}{r} + \frac{ar_0^2}{3r^3},$$

where a and r_0 are positive constants. When disturbed slightly from its stable equilibrium position, it undergoes a simple harmonic oscillation. The time period of oscillation is

(A) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ (B) $2\pi\sqrt{\frac{mr_0^3}{a}}$ (C) $2\pi\sqrt{\frac{2mr_0^3}{a}}$ (D) $4\pi\sqrt{\frac{mr_0^3}{a}}$

13. A planet of mass m moves in a circular orbit of radius r_0 in the gravitational potential $V(r) = -\frac{k}{r}$, where k is a positive constant. The orbital angular momentum of the planet is

(A) $2r_0km$ (B) $\sqrt{2r_0km}$ (C) r_0km (D) $\sqrt{r_0km}$

14. The solution of the differential equation

$$\frac{d^2y}{dt^2} - y = 0,$$

subject to the boundary conditions y(0) = 1 and y(∞) = 0, is

(A) cos t + sin t (B) cosh t + sinh t

(C) cos t - sin t (D) cosh t - sinh t

15. In an interference pattern formed by two coherent sources, the maximum and the minimum of the intensities are $9I_0$ and I_0, respectively. The intensities of the individual waves are

(A) $3I_0$ and I_0 (B) $4I_0$ and I_0 (C) $5I_0$ and $4I_0$ (D) $9I_0$ and I_0

16. Identify the CORRECT statement for the following vectors $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$

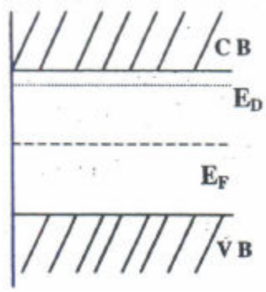
(A) The vectors $\vec{a}$ and $\vec{b}$ are linearly independent

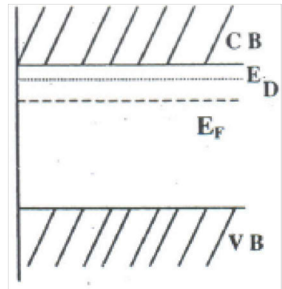
(B) The vectors $\vec{a}$ and $\vec{b}$ are linearly dependent

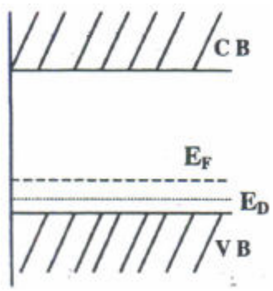
(C) The vectors $\vec{a}$ and $\vec{b}$ are orthogonal

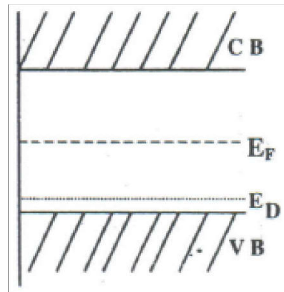
(D) The vectors $\vec{a}$ and $\vec{b}$ are normalized

17. Identify, the CORRECT energy band diagram for Silicon doped with Arsenic. Here CB, VB, E_D and E_F are conduction band, valence band, impurity level and Fermi level, respectively.

(A) 

(B) 

(C) 

(D) 

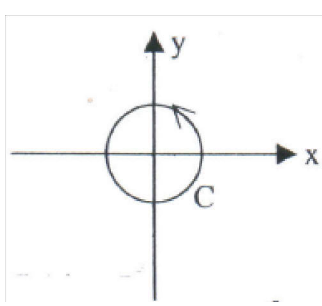
18. A pendulum suspended from the ceiling of the train is at rest. What will be the time period of pendulum if the train accelerates at 10 m/s^2 (Take g = 10 m/s^2)

(A) 2S (B) $2\sqrt{2}$ S (C) $2/\sqrt{2}$ S (D) $\frac{2}{2^{1/4}}$ Sec

19. A rod of proper length l_0 oriented parallel to the x-axis moves with speed 2c/3 along the x-axis in the. S-frame, where c is the speed of the light in free space. The observer is also moving along the x-axis with speed c/2 with respect to the S-frame. The length of the rod as measured by the observer is

(A) 0.35 l_0 (B) 0.48 l_0 (C) 0.87 l_0 (D) 0.97 l_0

20. Given $\vec{F} = \vec{r} \times \vec{B}$, where $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})$ is a constant vector and r is the position vector. The value of $\oint_C \vec{F} \cdot d\vec{r}$, where C is a circle of unit radius centered at origin is _____.



(A) 0 (B) $2\pi B_0$ (C) $-2\pi B_0$ (D) 1

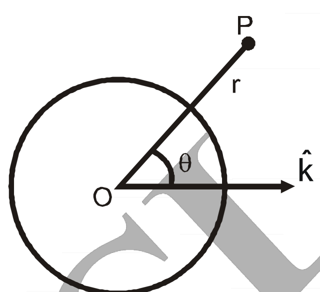
21. A system has two energy levels with energies ε and 2ε. The lower is 4-fold degenerate while the upper level is doubly degenerate. If there are N non-interacting classical particles in the system, which is in thermodynamic equilibrium at a temperature T, the fraction of particles in the upper level is

(A) $\frac{1}{1+e^{-\varepsilon/k_BT}}$ (B) $\frac{1}{1+2e^{\varepsilon/k_BT}}$ (C) $\frac{1}{2e^{\varepsilon/k_BT}+4e^{2\varepsilon/k_BT}}$ (D) $\frac{1}{2e^{\varepsilon/k_BT}-4e^{2\varepsilon/k_BT}}$

22. A spherical conductor of radius a is placed in a uniform electric field $\vec{E}=E_0\hat{k}$. the potential at a point P(r, θ) for r > a, is given by

$$f(r, \theta) = \text{constant} - E_0 r \cos\theta + \frac{E_0 a^3}{r^2}\cos\theta$$

where r is the distance of P from the centre O of the sphere and q is the angle OP makes with the z-axis.



The charge density on the sphere at q = 30° is

(A) $3\sqrt{3}\varepsilon_0E_0/2$ (B) $3E_0E_0 / 2$

(C) $\sqrt{3}\varepsilon_0E_0/2$ (D) $E_0E_0 / 2$

23. The solution of the differential equation for $y(t): \frac{d^2y}{dt^2}-y=2\cosh(t),$ subject to the initial conditions $y(0)=0$ and $\left.\frac{dy}{dt}\right|_{t=0}=0,$ is

(A) $\frac{1}{2}\cosh(t)+t\sinh(t)$ (B) – sin h (t) + t cosh(t)

(C) t cosh(t) (D) t sinh (t)

24. A particle of mass m is confined in the potential

$$v(x)=\begin{cases}\frac{1}{2}m\omega^2x^2 & \text{for } x>0,\\ \infty & \text{for } x\le 0.\end{cases}$$

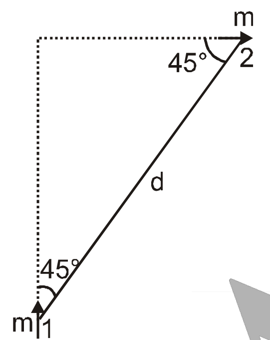
Let the wave function of the particle be given by

$$\psi(x) = -\frac{1}{\sqrt{5}}\psi_0 + \frac{2}{\sqrt{5}}\psi_1,$$

where y_0 and y_1 are the eigen functions of the ground state and the first excited state respectively. The expectation value of the energy is

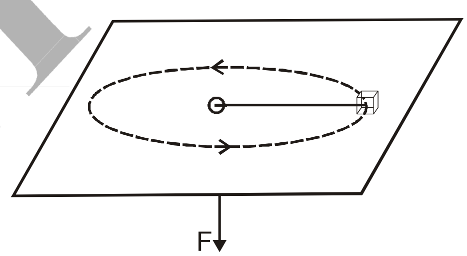
(A) $\frac{31}{10}\hbar\omega$ (B) $\frac{25}{10}\hbar\omega$ (C) $\frac{13}{10}\hbar\omega$ (D) $\frac{11}{10}\hbar\omega$

25. Two magnetic dipoles of magnitude m each placed in a plane as shown. The energy of interaction is given by



(A) Zero (B) $\frac{\mu_0}{4\pi}\frac{m^2}{d^3}$ (C) $\frac{\mu_0}{2\pi}\frac{m^2}{d^3}$ (D) $-\frac{3\mu_0}{8\pi}\frac{m^2}{d^3}$

26. A mass m is constrained to move on a horizontal frictionless surface. It is set in circular motion with radius r_0 and angular speed w_0 by an applied force $\vec{F}$ communicated through an inextensible thread that passes through a hole on the surface as shown in the figure. This force is then suddenly doubled. The magnitude of the radial velocity of the mass



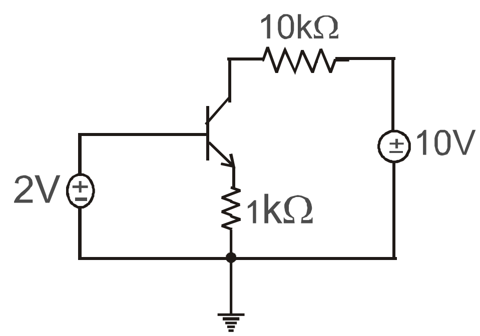
(A) Increases till the mass falls into the hole.

(B) Decreases till the mass falls into the hole.

(C) remains constant.

(D) Becomes zero at a radius r_1 where $0 < r_1 < r_0$

27. For the BJT circuit shown, assume that the β of the transistor is very large and V_{BE} = 0.7V. The mode of operation of the BJT is ___



(A) Cut-off (B) Saturation

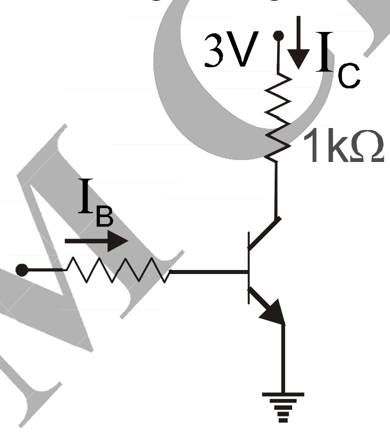
(C) Birnak active (D) Reverse active

28. If for a silicon n-p-n transistor, the base-to-emitter voltage (V_{BE}) is 0.7V and the collector-to-base voltage (V_{CB}) is 0.2V, then the transistor is operating in the___

(A) Normal active mode (B) Saturation mode

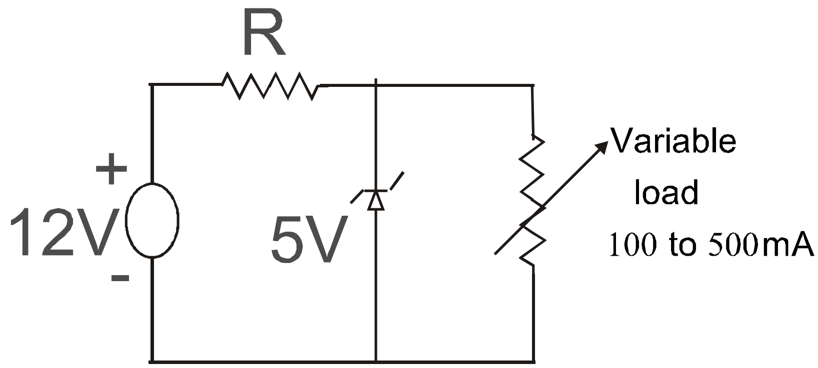
(C) Inverse active mode (D) Cut-off mode

29. Assuming V_{CEsat} = 0.2V and β = 50, the minimum base current (I_B) required to drive the transistor in the given figure to saturation is___



(A) 56 μA (B) 140mA (C) 60 μA (D) 3 mA

30. In the voltage regulator shown in the given figure, the load current can vary from 100 mA to 500 mA. Assuming that the Zener diode is ideal (i.e, the Zener knee current is negligibly small and Zener resistance is zero in the breakdown region), the value of R is ____



(A) 7Ω (B) 70Ω (C) $\frac{70}{3}\Omega$ (D) 14Ω

SECTION-B (Q.31-40) MULTIPLE SELECT QUESTIONs (MSQs) :

31. A converging lens is used to form an image on screen. When the upper half of the lens is covered by an opaque screen.

(A) half of the image will disappear

(B) complete image will be formed.

(C) intensity of the image will increase.

(D) intensity of the image will decrease.

32. A beam of light of wavelength 400 nm and power 1.55 mQ is directed at the cathode of a photoelectric cell. (given hc = 1240 ev. nm If only 10 % of the current due to these electrons is I. If the wavelength of light is now reduced to 200 nm, keeping its power the same, the kinetic energy of the electrons is found to increase by a factor of 5. The stopping potentials for the two wavelength are S_1 and S_2. Then

(A) $I = 10$mA (B) I=100 mA (C) S_1 = 0.775V (D) S_2 = 3.875V

33. In a compton experiment the ultraviolet light of wavelength 2000 A$^\circ$ is scattered from an electron at rest. What should be the minimum resolving power of an optical instrument to measure the compton shift. If the observation is made at 60° and 90° with respect to the direction of the incident light ?

(A) R_{60} = 164880 (B) R_{60} = 164600

(C) R_{90} = 81300 (D) R_{90} = 82440

34. X-Rays of wavelength $\lambda = 0.612\,A^\circ$ are scattered by free electron (compton collision). Then which of the following statements is/are correct -

(A) The wave length of scattered radiation at an 90° with the direction of incidence is $0.6362\,A^\circ$.

(B) Kinetic energy imparted to the electron is 7.72 eV.

(C) Kinetic energy imparted to the electron is 2.02 eV.

(D) Compton collision is elastic collision.

35. A particle of mass m and energy E moving in the positive x - direction, encounters a one - dimensional potential barrier at x = 0. The barrier is defined by

$$\begin{matrix} v = 0 \text{ for } x < 0 \\ v = v_0 \text{ for } x > 0 \end{matrix} \begin{cases} v_0 > 0 \\ E > v_0 \end{cases}$$

If the wave function of the particle in the region x < 0 is given as $Ae^{ikx} + Be^{-ikx}$. If $\frac{B}{A} = 0.4$, then

(A) $\frac{E}{v_0} = 1.225$ (B) T = 0.16 (C) R = 0.84 (D) R + T = 1

36. A resistor of $1K\Omega$ and an inductor of 5mH are connected in series with a battery of emf 4v through a switch. The switch is closed at time t = 0. (use $e^3 = 20$)

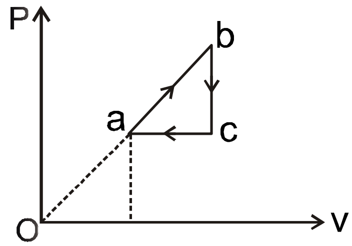
(A) The current flowing in the circuit at $t = 15\mu\text{sec}$ is 3.8 mA

(B) The current flowing in the circuit at $t = 15\mu\text{sec}$ is 4 mA

(C) The heat dissipated through the resister during the first $15\mu\text{sec}$ is 12.8 x 10^{-10} J.

(D) The heat dissipated through the resister during the first $15\mu\text{sec}$ is 6.4×10^{10}J

37. One mole of an ideal gas (r = 1.5) is at pressure of 100 K P_a and a temperature of 300 K. Initial the state of the gas is at the point a of the PV diagram shown. The gas is taken through a reversible cycle $a \rightarrow b \rightarrow c \rightarrow a$. The pressure at point b is 200 K P_a. Then



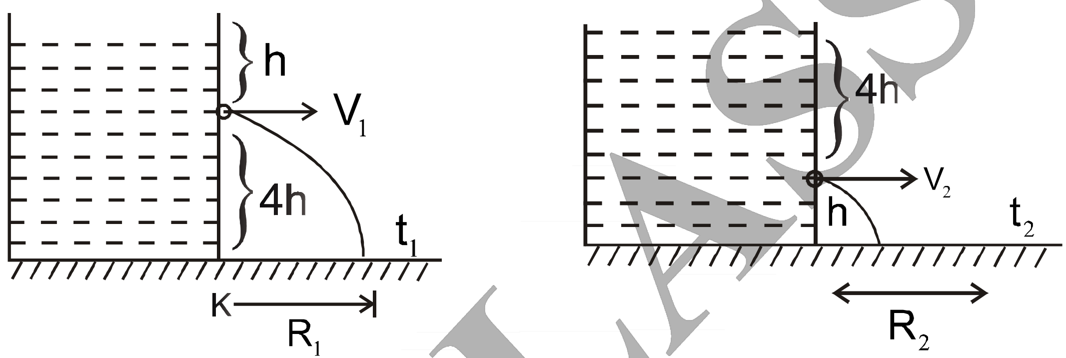
(A) W_{ab} = 50KJ

(B) W_{ac} = 100 KJ

(C) W_{bc} = 0

(D) $\Delta S_{ab} = 2\ell n^2$ KJ/K

38. In two figures



(A) $V_1/V_2 = \frac{1}{2}$

(B) $t_1/t_2 = 2$

(C) $\frac{R_1}{R_2} = 1$

(D) $\frac{V_1}{V_2} = \frac{1}{4}$

39. A coil of area $2m^2$ and resistance 4Ω is placed perpendicular to a uniform magnetic field of 4T. The loop is rotated by 90° in 0.1 sec. Choose the correct options-

(A) Average induced emf in the coil is 8V.

(B) Average induced current in the circuit is 20A.

(C) 2C charge will flow in the coil in above period.

(D) Heat produced in the coil in the above period can't be determined from the given data.

40. A Voltage 200 cos 100 t is applied to a half wave rectifier with load resistance of $5k\Omega$. Rectifier may be represented by an ideal diode in series with a resistance of $1k\Omega$.

Then which of the following statements is/are correct :-

(A) Rms value of current = 16.67 mA

(B) dc value of current = 10.61 mA

(C) Rectifier Efficiency is 40.6%

(d) Ripple factor = 1.21

SECTION-A (Q.41-60) NUMERICAL ANSWER TYPE (NATs) :

41. An unpolarized light wave is incident from air on a glass surface at the Brewster angle. Calculate the angle (in degree) between the reflected and the refracted wave.

42. y_1 and y_2 are two orthogonal states of a spin 1/2 system. It is given that

$$\psi_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix},$$

where $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ represent the spin-up and spin-down states, respectively.

When the system is in the state y_2, its probability to be in the spin-up state is ____________.

43. A one dimensional harmonic oscillator is in the superposition of number states, |n**ñ**, given by

$$|\psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle.$$

The average energy of the oscillator in the given state is __________hw.

44. A ball of mass m, initially at rest, is dropped from a height of 5 meters. If the coefficient or restitution is *0.9*, the speed of the ball just before it hits the floor the second time is approximately ________ m/s (take g = 9.8 m/s^2)

45. When an ideal monoatomic gas is expanded adiabatically from an initial volume V_0 to $3V_0$, its temperature changes from T_0 to T. Then the ratio $\frac{T}{T_0}$ is _____.

46. Let E_S denote the contribution of the surface energy per nucleon in the liquid drop model. The ratio $E_S\left({}^{27}_{13}Al\right) : E_S\left({}^{64}_{30}Zn\right)$ is _______.

47. A particle moves in a plane with velocity $\vec{v} = v_r\hat{r} + v_\theta\hat{\theta}$ such that $v_r = \frac{3}{4}v_\theta$. The time dependence of the magnitude of the velocity $|\vec{v}|$ = 5t. It is given that r = 1, θ =0 and v_r > 0 at t = 0. (In the following, you may use $e^3 \approx 20$.) At what time (in sec.) will θ become 4 radian?

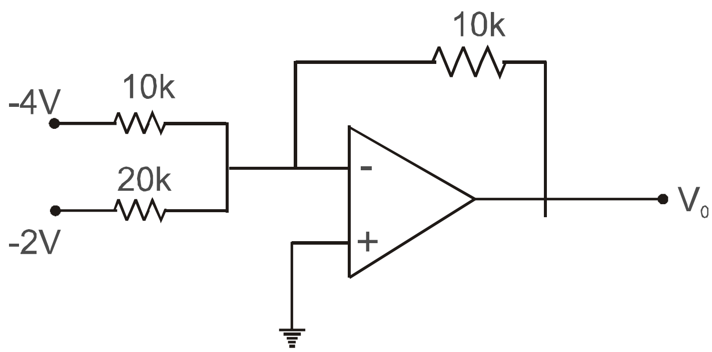
48. A body of mass 1 kg moves under the influence of a central force, with a potential energy function $V(r) = -\frac{\exp(-3r/2)}{5r^2}$ Joule, where r is in meters. It is found to move in a circular orbit of radius r = 2 m. (In the following, you may use $e^3 \approx 20$). Find its total energy E. (in mJ)

49. A cylinder of 1 kg mass and 0.02 m diameter left at the top of an inclined plane of height 1 m rolls down without slipping. (Take g = 9.8 ms^{-2}). Find the velocity of center of mass of cylinder on reaching the bottom of inclined plane (in m/sec).

50. X-Rays of wavelength λ = 0.612 x 10^{-10} m are scattered by free electrons (Compton collision). Calculate the corresponding kinetic energy in eV imparted to the electron.

51. A parallel beam of light of diameter 1.8 cm contains two wavelengths 4999.75 Å and 5000.25 Å. The light is incident perpendicularly on a large diffraction grating with 5000 lines per centimeter. Using Rayleigh criterion, find the least order at which the two wavelengths are resolved.

52. If differential gain of an opamp is A_{vd} = 3500 % & Common mode gain (A_{cM}) = 0.35, Then CMRR will be ______dB.

53. 

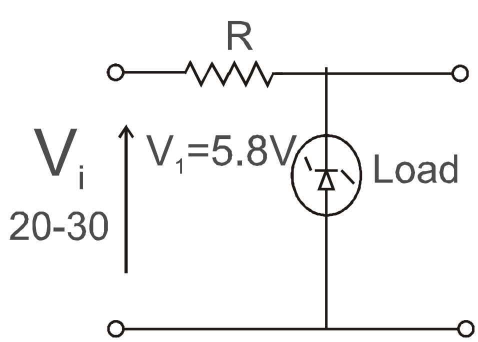
The output in the above circuit will be ______V.

54. An aircraft , which weights 12000 kg when unloaded , is on a relief mission , carrying 4000 food packets weighing 1 kg each. The plane is gliding horizontally with its engines off at a uniform speed of 540 kmph when the first food packet is dropped. Assume that the horizontal air drag can be neglected and the aircraft keeps moving horizontally. If one food packet is dropped every second , then the distance between the last two packet drops will be _______ m.

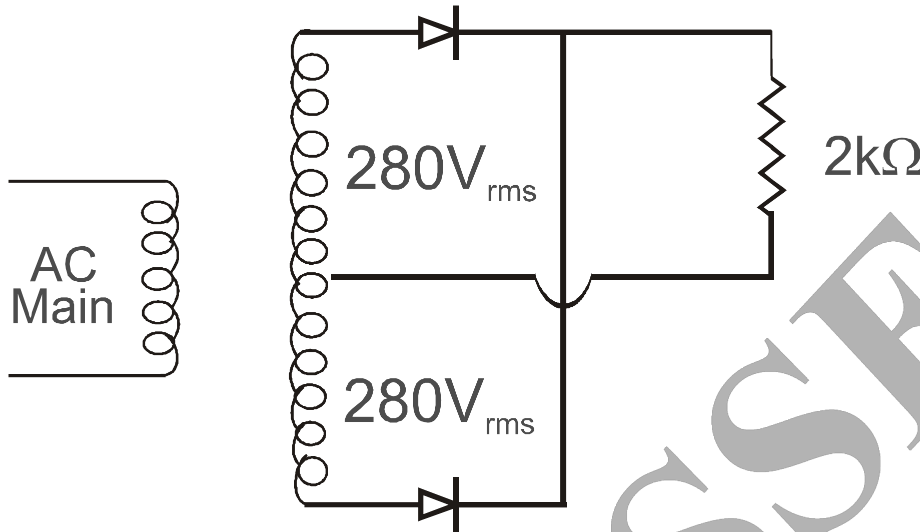
55. In the temperature range 100 - 1000 C, the molar specific heat of a metal varies with temperature T (measured in degrees Celsius) according to the formula C_p=(1 + T/ 5) J-deg C^{-1}-mol^{-1}. If 0.2 Kg of the metal at 300 C, the final equilibrium temperature, in degree C, will be ________.
[Assume that no heat is lost to radiation and /or other effects .]

56. On a planet having the same mass and diameter as the Earth, it is observed that objects weightless at the equator. Find the time period of rotation of this planet in minutes (as defined on the Earth).

57. The Zener diode in the regulator circuit shown in the figure has a Zener voltage of 5.8 Volts and a Zener knee current of 0.5 mA. The maximum load current drawn from this circuit ensuring proper functioning over the input voltage range between 20 and 30 volts, is _____ mA.

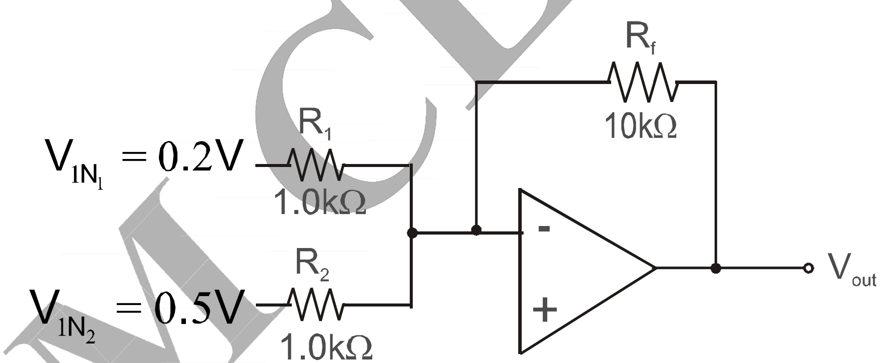


58. The centre tap full wave single-phase rectifier circuit uses 2 diodes as shown in the given figure. The rms voltage across each diode is _______ V.



59. If $\alpha = 0.98, I_{C0} = 6\mu A, I_{\beta} = 100\mu A$ for a transistor, than the value of I_C will be ____ mA.

60. The output voltage for the summing amplifier shown in figure is ______ V.



ANSWER KEY FMTP

Ques	1	2	3	4	5	6	7	8	9	10
Ans	C	B	C	D	B	B	C	D	A	B
Ques	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Ans	A	A	D	D	B	A	B	D	D	C
Ques	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans	B	A	D	C	D	B	B	A	A	D
Ques	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
Ans	B	C,D	A,D	A,B,D	A,D	A,C	A,B,C,D	A,B,C	B,C,D	A,B,D
Ques	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
Ans	90	0.67	3.25	8.91	0.481	1.333	3.55	-3.75	3.6	772.55
Ques	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans	2	80	3	150	519	84.7	24.2	395.3	5.2	-7

HINTS & SOLUTION

1.(C) Two macroscopic system, which are in thermal contact, the required condition for the equilibrium is that their b parameter must be equal.

$b = \frac{\partial}{\partial E}\log_e G$, G ® accessible phase space volume

So $\frac{1}{\beta} = kT$

$$\frac{1}{T} = k\frac{\partial}{\partial E}\ln G$$

$$\partial S = \frac{\partial Q}{T}$$

$$\frac{\partial S}{\partial Q} = \frac{1}{T}$$

$$\frac{\partial S}{\partial Q} = k\frac{\partial}{\partial E}(k \ln G)$$

on comparing

$$\boxed{S = k\ln\Gamma(E, V, N)}$$

and it is obvious that at equilibrium the accessible phase volume is maximum.

2.(B) The first law of thermodynamics can be given

$$dQ = dU + dW$$

here dQ and dW both are path function and dU is state function. For making dW a perfect differential, we take dQ = 0

So $\quad dU + dW = 0$

$\qquad dW = -dU$

and dQ = 0 occur in adiabatic process, so, dW_0 is a perfect differential if the process is adiabatic.

3.(C) For finding the vector which is perpendicular to any vector on the plane is given by taking the gradient of that plane eq^n.

The plane is $\phi = x + y + z - 5$

New grad $\phi = \hat{i}\dfrac{\partial \phi}{\partial \phi} + \hat{j}\dfrac{\partial \phi}{\partial y} + \hat{k}\dfrac{\partial \phi}{\partial z}$

$= \hat{i}.1 + \hat{j}.1 + \hat{k}.1$

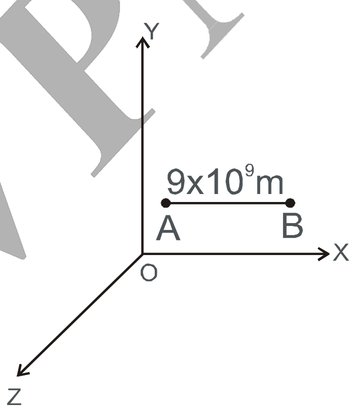
$\boxed{\text{grad } \phi = \hat{i} + \hat{j} + \hat{k}}$

4.(D) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$

$A - \lambda I = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 2 & 3 \\ 2 & 4-\lambda & 6 \\ 3 & 6 & 9-\lambda \end{vmatrix} = 0$

or $\qquad \lambda^2(14-\lambda) = 0 \quad$ or $\quad \lambda = 0, 0, 14$

5.(B)



Time measured at A, $t'_A = \dfrac{t_0 - vx_A / c^2}{\sqrt{1-\beta^2}}$

Time measured at B, $t'_B = \dfrac{t_0 - vx_B / c^2}{\sqrt{1-\beta^2}}$

$$t'_B - t'_A = \frac{v(x_A - x_B)}{c^2\sqrt{1-\beta^2}} = \frac{v\Delta x}{c^2\sqrt{1-\beta^2}}$$

$$\Delta t = \frac{v\Delta x}{c^2\sqrt{1-\beta^2}} = \frac{0.8c x 9x10^9}{\left(3x10^8\right)^2\sqrt{1-\frac{(0.8c)^2}{c^2}}}$$

$$\Delta t = \frac{0.8cx9x10^9}{9x10^{16}x0.6} = \frac{0.8x3x10^8x9x10^9}{9x10^{16}x0.6} = 40s.$$

6.(B) $\left[x^2, p^2\right]$

$= \left[x^2, pp\right] = p\left[x^2, p\right] + \left[x^2, p\right]p$

Since [A, BC] = B [A, C] + [A, B] C

So $\Rightarrow [A,B] = p[2i\hbar x] + [2i\hbar x]p$

$$\left[x^2, P^2\right] = 2i\hbar[px + xp]$$

7.(C) According to malus law the transmitted light intensity is proportional to the square of cosine of angle between polarizer and analyzer.

I $\cos^2\theta$ $\Rightarrow$ I = $I_0 < \cos^2\theta > = \dfrac{I_0}{2}$

when light say passing through P_1 the angle is q = 0° so

I = $I_0 \times \dfrac{1}{2}$ ...(A)

when this polarized light passing through P_2 which makes an angle q with P_1 and

P_2 so

$$I = \frac{I_0}{2}\cos^2\theta \qquad \text{...(B)}$$

Now P_1 and P_3 are 1 to each other so according to malus law the transmitted intensity

$$I = \frac{I_0}{2}\cos^2\theta\ \cos^2\frac{\pi}{2}\theta$$

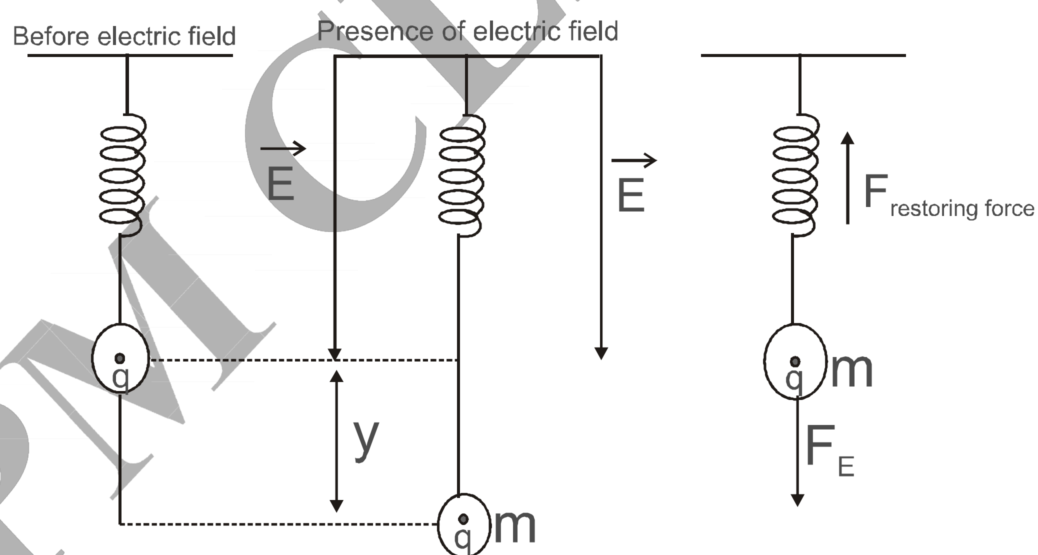
$$I = I_0\cos^2\theta\ \sin^2\theta$$

$$= \frac{I_0}{8} 4\cos^2\theta \sin^2\theta$$

$$\boxed{I = \frac{I_0}{8}\sin^2 2\theta}$$

8.(D)

Let, in presence of electric field, the spring elongates by; distance 'y'



The direction of motion is along y direction, then equation of motion is,

$$m\ddot{y} = F_E - F_{\text{restoring force}}$$

$$m\ddot{y} = qE - ky, \text{or}, m\ddot{y} + ky = qE, \text{or}, \ddot{y} + \frac{k}{m}y = \frac{qE}{m}$$

So, $\ddot{y} + \omega^2 y = \dfrac{qE}{m}$

To find C.F. $\frac{d^2y}{dt^2}+\omega^2 y=0$

$\left(D^2+\omega^2\right)y=0, \text{or}, D=\pm i\omega$

$y_{C.F}=c_1\cos\omega t+c_2\sin\omega t$

Now, P.I. =

$$\frac{1}{D^2+\omega^2}\left(\frac{qE}{m}\right)=\frac{1}{D^2+\omega^2}e^{0t}\left(\frac{qE}{m}\right)=\left(\frac{qE}{m}\right)\frac{1}{D^2+\omega^2}e^{0t}=\frac{qE}{m(0^2+\omega^2)}=\frac{qE}{m\omega^2}$$

So, y = $y_{C.F}+y_{P.I.}=C_1\cos\omega t+C_2\sin\omega t+\frac{qE}{m\omega^2}$

Now, apply boundary conditions, at t = 0, y = 0

So, $0=C_1+0+\frac{qE}{m\omega^2}, \text{or}, C_1=-\frac{qE}{m\omega^2}$.

$$y=-\frac{qE}{m\omega^2}\cos\omega t+\frac{qE}{m\omega^2}=\frac{qE}{m\omega^2}(1-\cos\omega t)=\frac{qE}{m\omega^2}(1-\cos\omega t)=\frac{qET}{m\omega}\left(\frac{(1-\cos\omega t)}{2\pi}\right);$$

where, $T=\frac{2\pi}{\omega}$.

Now, the amplitude of oscillation is, $y_0=\frac{qET}{m\omega}=\frac{qET}{\sqrt{mk}}$.

9.(A) $V_{cc}=\left(I_b+I_c\right)R_C+V_{ce}\approx I_cR_C+V_{be.}$

$$I_C=\frac{V_{cc}-V_{ce}}{R_c}=\frac{(20-4)}{1x10^3}=16x10^{-3}A=16mA$$

$$I_b=\frac{I_c}{\beta}=160\,\mu A$$

$$V_{ce}=I_bR_b+V_{be}\simeq I_bR_b$$

$$R_b = \frac{V_{ce}}{I_b} = \frac{4}{160\mu A} = 25x10^3\Omega = 25k\Omega$$

10. (B)

In a bipolar transistor, the increase in the depletion region of the reverse biased junction, tends to decrease the width of the base with increased reverse voltage. This in turn causes the current gain to increase with increased voltage resulting in a nearly linear increase in current even with the device in the saturation.

11.(A) The momentum conjugate to r is = $\frac{dm|r|}{dt} = m\frac{dr}{dt} = m\dot{r}$

12.(A) V(r) = $-\frac{a}{r} + \frac{ar_0^2}{3r^3}$

where a & r_0 is positive constant

$$\frac{\partial V}{\partial r} = \frac{a}{r^2} - \frac{3ar_0^2}{3r^4}$$

For stable equilibrium $\frac{\partial V}{\partial r}$ = 0

$$\frac{a}{r^2} = \frac{3ar_0^2}{3r^4} \quad \boxed{r^2 = r_0^2}$$

We know that the time period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{K}}$$

where K is force constant

and K = $\left.\frac{\partial^2 V}{\partial r^2}\right|_{\text{equilibrium position}}$

$$\frac{\partial^2 V}{\partial r^2} = -\frac{2a}{r^3} + \frac{4ar_0^2}{r^5} \qquad \text{at } r = r_0$$

$$= -\frac{2a}{r_0^3} + \frac{4ar_0^2}{r_0^5} = -\frac{2a}{r_0^3} + \left(\frac{4a}{r_0^3}\right) = \frac{2a}{r_0^3}$$

$$\boxed{T = 2\pi\sqrt{\frac{mr_0^3}{2a}}}$$

13.(D) Gravitational potential $V(r) = \frac{-k}{r}$

Gravitational Force (F) = $\frac{dV}{dr} = \frac{k}{r_0^2}$

$\frac{mv^2}{r_0} = \frac{k}{r_0^2} \Rightarrow v = \sqrt{\frac{k}{mr_0}}$

$\text{momentum } (p) = mv = \sqrt{\frac{mk}{r_0}}$

$\text{Anglax momentum}(L) = \vec{r} \times \vec{p} = r_0 p \sin 90^\circ = r_0 p$

$L = \sqrt{mkr_0}$

14.(D) The differential equation

$$\frac{d^2y}{dt^2} - y = 0$$

$(D^2 - 1)y = 0 \quad \Rightarrow m^2 - 1 = 0 \quad \Rightarrow m = \pm 1$

$y(t) = c_1e^{-t} + c_2e^{t}$ ....(i)

according to boundary conditions; $y(0) = 1$

$1 = c_1 + c_2$ ...(ii)

& $\quad y(\infty) = 0$

$0 = c_1e^{-\infty} + c_2e^{\infty}$

$0 = 0 + c_2e^{\infty}$ ...(iii)

equation 3 will be satisfy if or only if

$\mathbf{c_2 = 0}$

then $c_1 + c_2 = 1$

$c_1 = 1$

$y(t) = e^{-t}$

$y(t) = \frac{e^{-t}}{2} + \frac{e^{-t}}{2} + \frac{e^{t}}{2} - \frac{e^{-t}}{2}$ {adding and subtracting $e^t/2$}

$= \frac{e^{t} + e^{-t}}{2} - \frac{e^{t} - e^{-t}}{2}$

$\boxed{y(t) = \cosh t - \sinh t}$

15.(B) The max & min intensities by two coherent sources is given by:-

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = I_1 + I_2 + 2\sqrt{I_1 I_2} = 9I_0$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = I_1 + I_2 - 2\sqrt{I_1 I_2} = I_0$$

Solving the above two eq, we get

$$I_1 = 4I_0 \quad \& \quad I_2 = I_0$$

16.(A) The determinate is not equal to zero

$$\begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} = 6 - 4 \neq 0$$

$\vec{a}$ and $\vec{b}$ have no mutual relation so it is linearly independent.

17.(B) As doping with Arsenic Will change the pure silicon Semiconductor into n - type & in n - type SC the free charge carriers are present does to CB because Arsenic is a pentavalent atom thus the atom Thus the fermi Energy level will lie slightly above the middle of the Energy gap & does to E_D.

18.(D) Net acceleration = $(g^2 + a^2)^{1/2}$ = $((10)^2 + (10)^2)^{1/2}$ = $10\sqrt{2}$

So, $T' = 2\pi\sqrt{\frac{\ell}{g'}} = 2\pi\sqrt{\frac{\ell}{10\sqrt{2}}}$ ..(A)

For simple pendulum $T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell}{10}}$

T = 2 se. ..(B)

So, from eq.(A) and (B),

$$T' = \frac{T}{\left(\sqrt{2}\right)^{1/2}}$$

$$T' = \frac{2}{2^{1/4}} \text{ sec.}$$

19.(D) For finding the length of rod firstly finding the relativistic velocity which is equal to

$$u'_x = \frac{u_x - V}{1 - \frac{u_x V}{c^2}}$$

here $u_n = \frac{2c}{3}, V = \frac{c}{2}$

Now $u'_x = \dfrac{\frac{2c}{3} - \frac{c}{2}}{1 - \frac{2\cancel{c}}{3}, x \frac{\cancel{c}}{\cancel{2}} x \frac{1}{\cancel{c}^2}} \Rightarrow \dfrac{\frac{C}{6}}{1 - \frac{1}{3}}$

$$u'_x = \frac{\frac{C}{6}}{2/3}$$

$$u'_x = \frac{c}{4}$$

Now the measured length

$$L = \ell_0\sqrt{1 - \frac{u_x'^2}{c_2}} = \ell_0\sqrt{1 - \frac{c^2}{16c^2}} = \ell_0\sqrt{\frac{15}{16}} = .866\,\ell_0 \;= 0.97\ l_{0.}$$

20.(C) $\oint F.dr = \int \nabla \times F\ ds$

$$\Rightarrow \int\left[\nabla \times (r \times B) \cdot ds\right]$$

$$r \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ B_0 & B_0 & B_0 \end{vmatrix} \Rightarrow B_0\left[\hat{i}(y - z) + \hat{j}(z - x) + \hat{k}(x - y)\right]$$

Then $\nabla \times (r \times B) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y - z & z - x & x - y \end{vmatrix}$

$$B_0\left[\hat{i}\left[\frac{\partial}{\partial y}(x-y)-\frac{\partial}{\partial z}(z-x)\right]+\hat{j}\left[\frac{\partial}{\partial z}(y-z)-\frac{\partial}{\partial x}(x-y)\right]+\hat{k}\left[\frac{\partial}{\partial x}(z-x)-\frac{\partial}{\partial y}(y-z)\right]\right]$$

$$\int\nabla\times(r\times B)\cdot dS = B_0\left[\hat{i}[-2]-2\hat{j}-2\hat{k}=-2B_0(\hat{i}+\hat{j}+\hat{k})\right]$$

and $\oint ds = \pi a^2$

a = 1 then

$$\int\nabla\times(r\times B)\cdot ds = -2B_0\pi$$

21.(B) $E_1 = \varepsilon$ $E_2 = 2\varepsilon$

$N_1 = 4$ $N_2 = 2$

So, partition function $Z = \sum_i e^{-\beta\epsilon}$

$$z = 4e^{-\beta\varepsilon} + 2e^{-2\beta\varepsilon}$$

where $\beta = \frac{1}{KT}$

so, fraction of particles in the upper level is

$$P = \frac{2\exp(-2\in\beta)}{4e^{-\beta\epsilon}+2e^{-2\beta\epsilon}}$$

$$P = \frac{1}{2\exp(\beta\in)+1}$$

$$P = \frac{1}{1+2e^{\varepsilon/K_BT}}$$

22.(A) Potential f (r, θ) = const. – $E_0 r\cos\theta + \frac{E_0a^3\cos\theta}{r^2}$ ..(A)

We know electric field due to conducting sphere is

$$E = \frac{\sigma}{\epsilon_0}$$

$\Rightarrow$ s = $E\hat{I}_0$

$$E = -\nabla V = -\frac{\partial\phi}{\partial r} = +E_0\cos\theta - E_0 a^3\cos\theta\left[\frac{-2}{r^3}\right]$$

$$E = E_0\cos\theta + 2E_0\frac{a^3}{r^3}\cos\theta$$

E F. at the surface sphere of sphere and at θ = 30°

$$E/_{r=a/\theta=30^o} = E_0 \cos 30 + 2E_0 \frac{a^3}{a^3} \cos 30 = \frac{\sqrt{3}}{2}E_0 + 2E_0\frac{\sqrt{3}}{2}$$

$$E = \frac{3\sqrt{3}E_0}{2}$$

So, surface charge density s = Î$_0$E

$$\boxed{\sigma = \frac{3\sqrt{3}}{2} \in_0 E_0}$$

23.(D) $\frac{d^2y}{dt^2} - y = 2\cos h(t) = \left[\frac{\exp(t) + \exp(-t)}{2}\right]$

$(D^2 - 1)\, y = \exp(+) + \exp(-t)$

C.F. $\Rightarrow$ $D^2 - 1 = 0$

$0 = \pm 1$

i.e. C.F. = $C_1 e^t + C_2 e^{-t}$ ..(A)

$$\text{P.I.} = \frac{\left[e^t + e^{-t}\right]}{(D+1)(D-1)} = \frac{e^t}{(D+1)(D-1)} + \frac{e^{-t}}{(D+1)(D-1)} = \frac{1}{2}\frac{e^t}{(D-1)} - \frac{1}{2}\frac{e^{-t}}{(D+1)}$$

$$\text{P.I.} = \frac{1}{2}te^t - \frac{1}{2}te^{-t} \quad ..(B)$$

So, Total solution y(t) = C.F. + P.I.

$y(t) = c_1 e^t + c_2 e^{-t} + t \sin h\, t$ ..(C)

Given t = 0 , y = 0

$0 = C_1 + C_2 + 0$ Þ $C_1 + C_2 = 0$ ...(D)

$t = 0$, $\left.\frac{dy}{dt}\right|_{t=0} = 0$

$\frac{dy}{dt} = [c_1 et - c_2 e^{-t} + t \cosh t + \sin h\, t]_{t=0} = 0$

$= c_1 - c_2 + 0 + 0 = 0$

$\Rightarrow c_1 = c_2$ ...(5)

From equation (D) & (5) $\boxed{C_1 = 0,\ C_2 = 0}$

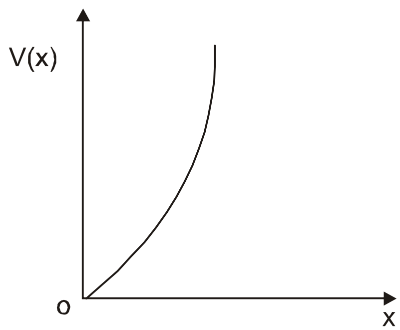
So, solution of differential equation

$$\boxed{y(t) = t \sinh(t)}$$

24.(C)
$$v(x) = \begin{cases} \frac{1}{2}m\omega^2 x^2 & \text{for } x > 0 \\ \infty & \text{for } x \le 0 \end{cases}$$

$$\psi(x) = \frac{-1}{\sqrt{5}}\psi_0 + \frac{2}{\sqrt{5}}\psi_1$$

y_0 ground state eigen function y_1® first excited state eigen function energy eigen value E =(n+1/2)$\hbar\omega$



for ground state n = 0, E = $\frac{1}{2}\hbar\omega$

for first excited state n =1, E = $\frac{3}{2}\hbar\omega$

$$\text{So,} \langle E \rangle = \int \psi E \psi d\tau = \langle \psi | E | \psi \rangle = \frac{-1}{\sqrt{5}} x \frac{-1}{\sqrt{5}} x \frac{1}{2}\hbar\omega \langle \psi_0 | \psi_0 \rangle + \frac{2}{\sqrt{5}} x \frac{2}{\sqrt{5}} x \frac{3}{2}\hbar\omega \langle \psi_1 | \psi_1 \rangle$$

$$\langle E \rangle = \left[\frac{1}{10} + \frac{12}{10}\right]\hbar\omega$$

$$\boxed{\langle E \rangle = \frac{13}{10}\hbar\omega}$$

25.(D) Angle between dipoles = 90°

Angles made by diploes with position vector (d) = 45 °

Magnetic interaction energy

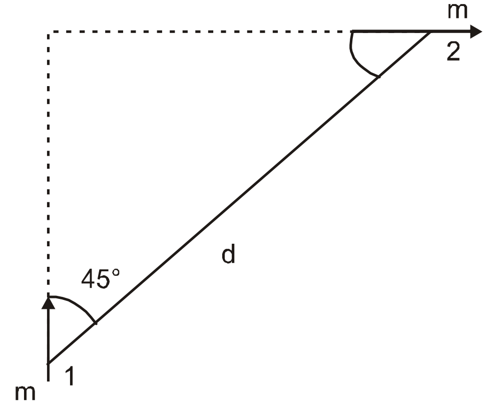
U = M.B

$$= -\frac{\mu_0}{4\pi}\left[\frac{3(m\bullet\hat{r})(m\bullet\hat{r})}{r^5} - \frac{m\bullet m}{r^3}\right] r \to d$$

$$U = -\frac{\mu_0}{4\pi}\left[\frac{3(mr\cos 45°)(mr\cos 45°)}{ds} - \frac{mm\cos 90°}{d^3}\right]$$

$$= -\frac{\mu_0}{4\pi}\frac{1}{d_5}\left[3m^2r^2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right]_{r\to d}$$

$$\boxed{U = \frac{-3\mu_0}{8\pi}\frac{m^2}{d^3}}$$



26.(B)

The particle is moving in a circular motion with radius r_0 and angular speed ω_0 by an applied force F applied through an inextensible thread passing through hole.

The circular motion of the particle is, $F_0 = \frac{mv_0^2}{r_0}$

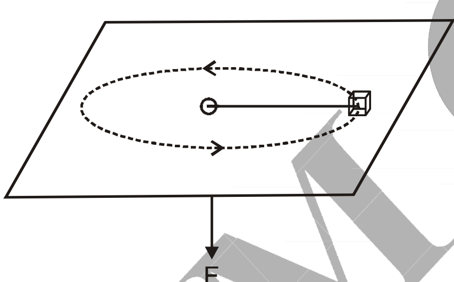
Now, when we doubled the force,

$2F_0$ = $2F_0 = \frac{mv^2}{r_0}$

Subtract equation (A) from (B), we get

$$F_0 = \frac{mv^2}{r} - \frac{mv_0^2}{r_0}$$

$$\frac{mv^2}{r} = F_0 + \frac{mv_0^2}{r_0}$$



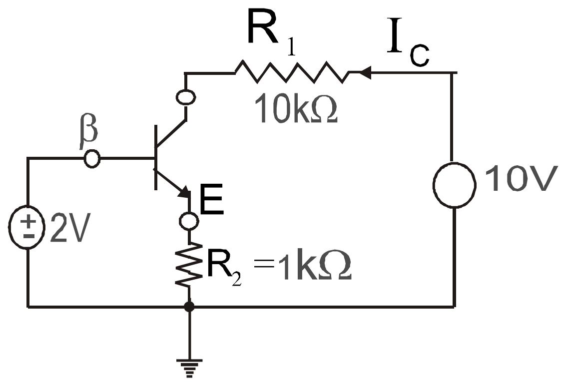
$$v = \left[\frac{r}{m}\left(F_0 + \frac{mv_0^2}{r_0}\right)\right]^{1/2}$$

As the force increases, r will decrease and hence, velocity of particle will decrease till the fall into the hole.

27. (B)

For saturation, V_{BE} > 0.7 volts In saturation condition,

$$I_C = \frac{V_{CC}}{R_1 + R_2} = \frac{10}{10+1} = \frac{10}{11}$$



Apply KVL in base circuit

$V_{BB} = V_{BE} + I_C R_2$

$$2 = V_{BE} + \left(\frac{10}{11}\right)$$

$\Rightarrow V_{BE} = 1.017$

So, operate in saturation region.

28. (A)

$V_B = 0.7V$ [Forward-biased]

and $V_{BC} = -V_{CB} = -0.2V$ (Reverse-biased)

Hence, the transistor operates normal active mode.

29. (A)

The minimum value of base current to derive the transistor into saturation is

$$(I_B)_{min} = \frac{I_C}{h_{fE}}$$

From the figure given,

$$I_C = \frac{3 - V_{CE,sat}}{1k\Omega} = \frac{30 - 0.2}{1k\Omega}$$

$$= 2.8 \text{ mA}$$

Then, $(I_B)_{min} = \dfrac{2.8\text{mA}}{50} = 56\mu A$

30. (D)

From the circuit given, we have

$$\frac{12 - V_Z}{R} = I_L + I_Z$$

or, $$I_L = \left(\frac{12 - V_Z}{R}\right) - I_Z$$

Given : 100 mA < I_L < 500 mA

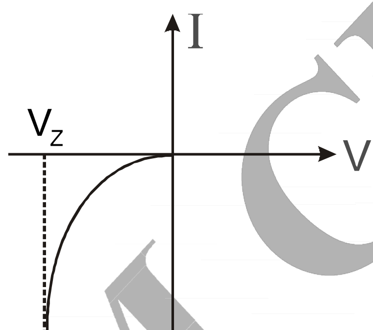
then $100 \times 10^{-3} < \left(\frac{7}{R} - I_Z\right) < 500 \times 10^{-3}$

$$100 \times 10^{-3} < \frac{7}{R} < 500 \times 10^{-3}$$

(Zener knee current is negligibly small)

or, $14\Omega < R < 70\Omega$

Hence, required value of R = 14Ω [closed min value]



31.(B)

When upper half of the lens is covered image is formed by the rays coming from lower half of the lens. OR image will be formed by the less number of rays. Therefore, intensity of image will decrease. But complete image will be formed.

32.(C,D)

Wavelength of incident light $\lambda = 400$nm

P = 1.55 mW

Energy of one photon $E = \frac{hc}{\lambda} = \frac{1240\,\text{eVnm}}{400\,\text{nm}} = 3.1\text{eV}$

Number of photons falling per second

$$n = \frac{P}{E} = \frac{1.55\text{mW}}{3.1\text{eV}} = \frac{1.55 \times 10^{-3}}{3.1 \times 1.6 \times 10^{-19}}$$

Number of electrons ejected is 10% of number of photon falling per second.

N = 0.1 n

$$= \frac{1.55 \times 10^{-4}}{3.1 \times 1.6 \times 10^{-19}} \text{ Per second.}$$

Current constipated by flow of ejected electrons

$$I = Ne = \frac{1.55 \times 10^{-4}}{3.1} = 50\text{mA}$$

Kinetic theory of ejected electron = Photon - Work function Energy of incident

$$\Rightarrow KE = \frac{hc}{\lambda_o} - W \quad \_\_\_\_\_\_\_\_\_(A)$$

Where $\lambda = 400\text{nm}$ is reduced to $\lambda = 200\text{nm}$, the KE increases by 5 times.

$$5KE = \frac{hc}{\lambda/2} - W \quad \_\_\_\_\_\_\_\_\_\_(B)$$

Subtracting equation (A) from (B)

$$4KE = \frac{hc}{\lambda} = 3.1\text{eV}$$

KE = 0.775 eV

Stopping potential

$$S_1 = V_s = \frac{KE}{e} = 0.775 \quad (\text{for } \lambda = 400\text{nm})$$

for $\lambda = 200\text{nm}$

KE = 5 x 0.775 eV = 3.875 eV

Stopping potential

$$S_2 = V_s = \frac{KE}{e} = 3.875\,V$$

33.(A,D)

For uv light of wavelength $2000\ A^\circ$,

Scallered at an angle θ

$$\Delta\lambda = \frac{h}{m_o c}(1-\cos\theta)$$

for $\theta = 90^\circ$

$$\Delta\lambda = \frac{h}{m_o c} = 0.02426\ A^\circ$$

and for $\theta = 60^\circ$

$$\Delta\lambda = \frac{h}{m_o c} = (1-\cos 60^\circ)$$

$$\Delta\lambda = 0.01213\ A^\circ$$

Minimum resolving power of instrument required to measure the compton effect to measure the compton effect

$$R = \frac{\lambda}{\Delta\lambda}$$

$$R_{90} = \frac{2000}{0.02426} = 82440$$

$$R_{60} = \frac{2000}{0.01213} = 164880$$

34.(A,B,D)

Wavelength of scattered radiation

$$\lambda' = \lambda + \frac{h}{m_o c}(1-\cos\phi)$$

$$= 0.612 \times 10^{-10} + 2.426 \times 10^{-12} \, (1-\cos\phi)\{\phi = 90^\circ\}$$

$$\lambda' = 0.6362 \times 10^{-10} \text{m}$$

Kinetic energy imparted to the electron

$$KE = \frac{hc}{\lambda} - \frac{hc}{\lambda'}$$

$$KE = \frac{1240\,\text{ev} - \text{nm}}{6.12\text{nm}} - \frac{1240\,\text{ev} - \text{nm}}{6.3626\,\text{nm}}$$

KE = 202.614 - 194.89

KE = 7.724 ev

35.(A,D)

Assuming region (I) for x < 0 and region (II) for x > 0 for region (I), wave function

$$\psi_I = Ae^{ik_1x} + Be^{-ik_1x}$$

Where $k_1 = \dfrac{\sqrt{2mE}}{h}$

For region (II), we can write wave function as

$$\psi_{II} = Ce^{ik_2x} \forall k_2 = \frac{\sqrt{2m(E-V_0)}}{\hbar}$$

At the boundary , x = 0

$$\psi_I = \psi_{II} \Rightarrow A + B = C$$

$$\psi_I' = \psi_{II}' = A - B = \frac{k_2}{k_1}c$$

$$\Rightarrow \frac{A+B}{A-B} = \frac{k_1}{k_2} \Rightarrow \frac{B}{A} = \frac{k_1 - k_2}{k_1 + k_2}$$

$$\Rightarrow \frac{B}{A} = \frac{\sqrt{2mE} - \sqrt{2m(E - V_0)}}{\sqrt{2mE} + \sqrt{2m(E - V_0)}} = \frac{\sqrt{E} - \sqrt{E - V_0}}{\sqrt{E} + \sqrt{E - V_0}}$$

According to the question

$$\frac{B}{A} = 0.4$$

$$\Rightarrow \frac{\sqrt{E} - \sqrt{E - V_0}}{\sqrt{E} + \sqrt{E - V_0}} = 0.4 \Rightarrow 0.4\left(\sqrt{E} + \sqrt{E - V_0}\right) = \sqrt{E} - \sqrt{E - V_0}$$

$$\Rightarrow 0.6\sqrt{E} = 1.4\sqrt{E - V_0}$$

$$\Rightarrow \frac{\sqrt{E - V_0}}{E} = \frac{0.6}{1.4} = \frac{3}{7}$$

$$\Rightarrow 49(E - V_0) = 9E \Rightarrow \frac{E}{V_0} = \frac{49}{40} = 1.225$$

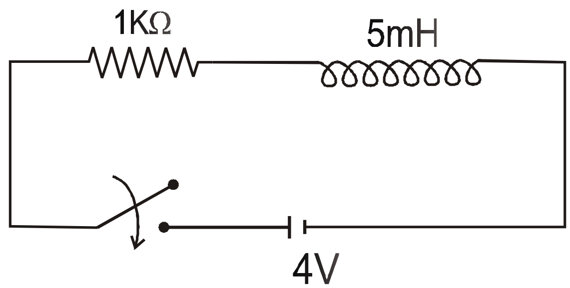
Reflection coefficient

$$R = \left|\frac{B}{A}\right|^2 = 0.16$$

Transmission coefficient

T = 1 - R = 0.84

36.(A,C)



When the switch is closed at t = 0. The circuit acts as charging R - L circuit. Instantaneous current flowing through the circuit can be given as

$$i = i_0\left(1 - e^{-\frac{t}{L/R}}\right)$$

$$i = i_0\left(1 - e^{\frac{tR}{L}}\right)$$

$i_0 \rightarrow$ steady state current

Putting t = 15 x 10^{-6} sec, L = 5 x 10^{-3} H and R = R $= 1K\Omega$

$$i = 4 \times 10^{-3}\left(1 - \exp\left(-\frac{10^3}{5\times 10^{-3}} \times 15 \times 10^{-6}\right)\right)$$

$$i = 4 \times 10^{-3}\,(1 - e^{-3})$$

$$i = 4 \times 10^{-3} \times \frac{19}{20} = 3.8\,\text{mA}$$

Again, heat dissipated through the resistor during first t second is

$$H = \int_0^t i^2 R\,dt = \int_0^t \left[i - \left(1 - e^{-\frac{R}{L}t}\right)\right]^2 R\,dt$$

$$= \int_0^t i_0^2 R\left(1 - e^{-\frac{R}{L}t}\right)^2 dt$$

$$= i_0^2 R\left[t - \frac{L}{2R}\left(e^{-\frac{2Rt}{L}} - 1\right) + \frac{2L}{R}\left(e^{-\frac{Rt}{L}} - 1\right)\right]$$

Putting t = 15μ sec , we will get

H = 12.799 x 10^{-10} J

37.(A,B,C,D)

For path $a \to b \Rightarrow p \,\alpha\, v$

$$\frac{P_a}{V_a} = \frac{P_b}{V_b}$$

$$\frac{(1000KP_a)}{1m^3} = \frac{200KP_a}{V_b}$$

Comparing with the process PV^{-x} = constant

$$W_{ab} = \frac{P_a V_b - P_b V_b}{-1-1} = 50KJ$$

During process bc, $\Delta W_{bc} = 0$

work done $\Delta W_{ca} = Pdv = 100x10^3$ $(1m^3)$ = 100KJ

again process ab

$$\Delta Q_{ab} = \Delta U_{ab} + \Delta W_{ab} = nc_v dt + PdV$$

$$ds = nc_v \frac{dT}{T} + \frac{PdV}{T}$$

$$\Delta S_{ab} = nc_v \int_{T_a}^{T_b} \frac{dT}{T} + nR\int_{T_a}^{T_b} \frac{dT}{T}$$

$$\Delta S_{ab} = 2\ell n^2 \, KJ/k$$

38.(A,B,C)

$$V = \sqrt{2gh_T}$$

$$t = \sqrt{\frac{2h_B}{g}}$$ and

$$R = 2\sqrt{h_T h_B}$$

Here h_T = distance of hole from top surface of liquid and h_B = distance of hole from bottom surface.

$$\Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{h}{4h}} = \frac{1}{2}$$

$$\Rightarrow \frac{t_1}{t_2} = \sqrt{\frac{4h}{h}} = 2$$

$$\Rightarrow \frac{R_1}{R_2} = \sqrt{\frac{4h.h}{h.4h}} = 1$$

39.(B,C,D)

$$\phi_i = BS\cos\theta = 4 \times 2 = 8\,Wb$$

$$\phi_f = BS\cos 90^\circ = 0$$

$$\Delta\phi = 8\text{ Wb}$$

$$|e| = \frac{\Delta\phi}{\Delta t} = \frac{8}{0.1} = 80\text{ Volt}$$

$$i = \frac{|e|}{R} = 20A$$

$$\Delta q = \frac{\Delta\phi}{R} = 2C$$

This current is not constant. So, we cannot find the heat generated unless current function with time is not known.

40.(A,B,D)

$$I_{max} = \frac{E_{max}}{R_F + R_L} = \frac{200}{1+5} = 33.33\,mA$$

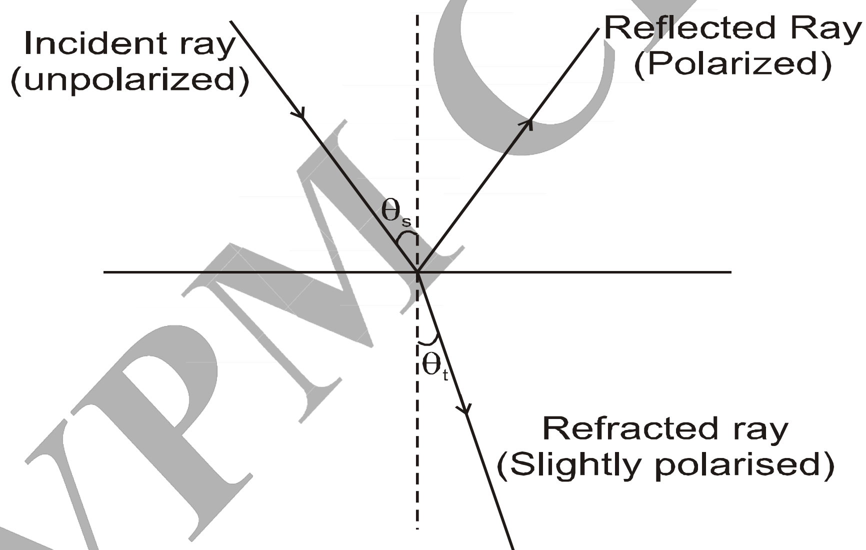
$$I_{dc} = \frac{I_{max}}{\pi} = 10.61\ mA$$

$$I_{rms} = \frac{I_m}{\pi} = \frac{33.33mA}{2} = 16.67\ mA$$

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{40.6}{1+\left(R_f/R_L\right)} = 33.83\ \%$$

r = 1.21

41. 90



Brewster angle is an angle of incidence at which light with a particular polarization is perfectly transmitted through a transparent dielectric surface with no reflection. When unpolarized light is incident at this angle the light that is reflected from the surface is therefore perfectly polarized.

Then the Brewster angle q_s

$\tan q_s = \frac{n_2}{n_1} = \frac{1.33}{1} = 53^o$

$q_s + q_t = 90^o$

and the angle between Reflected Ray and Refracted Ray will be 90º.

42. 0.67

$$y_1 = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix} + \sqrt{\frac{2}{3}}\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{\sqrt{3}}(\phi_1) + \frac{\sqrt{2}}{3}(\phi_2)$$

When the system is in state y_2, its probability to be in the spin up state is

$$= \left|\langle\phi_1|\psi_2\rangle\right|^2 = \frac{1}{3}\langle\phi_1|\phi_2\rangle + \frac{2}{3}\langle\phi_1|\phi_1\rangle = \frac{2}{3} = 0.67$$

43. 3.25

A one dimensional harmonic oscillator is in the superposition of number states |n**ñ**, given by

$$|\psi\rangle = \frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle.$$

Probability $P_1(E_1) = \frac{1}{4}$

$P_2(E_2) = \frac{3}{4}$

The energy of the |n**ñ** state in one dimensional harmonic oscillator

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

n = 2 $\quad E_1 = \frac{5}{2}\hbar\omega$

n = 3 $\quad E_2 = \frac{7}{2}\hbar\omega$

Total energy $\quad E = E_1P_1(E_1) + E_2P_2(E_2) = \left(\frac{1}{4}\times\frac{5}{2} + \frac{3}{4}\times\frac{7}{2}\right)\hbar\omega = \frac{5}{8} + \frac{21}{8} = \frac{26}{8}$

$= 3.25\ \hbar w$

44. 8.91

As we know that

$$\text{The coefficient of restitution} = \frac{\text{velocity of separation}}{\text{velocity of approaching}}$$

The velocity at the time of hitting with floor is = $\sqrt{2gh} = \sqrt{2 \times 9.8 \times 5}$

Velocity of approaching = $0.9\sqrt{2 \times 9.8 \times 5}$ = 8.91 m/sec

45. 0.4810

By the equation of adiabatic process

PV^r = constant

$T\,V^{r-1}$ = constant

$$T_0 V_0^{r-1} = T.(3V_0)^{r-1}$$

$$\Rightarrow \frac{T_0}{T} = (3)^{r-1} \qquad \left\{ \begin{array}{l} r \text{ for monoatomic} \\ \text{is } 5/3 \end{array} \right.$$

$$\Rightarrow \frac{T_0}{T} = (3)^{5/3-1} \qquad \Rightarrow \frac{T_0}{T} = (3)^{2/3} \qquad \Rightarrow \frac{T}{T_0} = \left(\frac{1}{3}\right)^{2/3} = 0.4810$$

46. 1.333

We know that semi empirical mass formula is

$$M(Z,A) = Zm_H + m_n(A\,Z) - a_1A + a_2A^{2/3} + a_3\,Z(Z-1)A^{-1/3} + a_4(Z - \frac{A}{2})^2 A^{-1} + a_5A^{-1/2}$$

$\forall$ Surface Energy $E_s = a_5A^{2/3}$

Surface Energy per nucleon is = $\dfrac{1}{A^{1/3}}$

$$\frac{E_s({}_{13}Al^{27})}{E_s\left({}^{64}_{30}\right)} = \left(\frac{64}{27}\right)^{1/3} = \frac{4}{3}$$

$\vec{v} = v_r\hat{r} + v_\theta\hat{\theta}$

$v_r = \frac{3}{4}V_\theta$

$\Rightarrow \frac{dr}{dt} = \frac{3}{4}r\frac{d\theta}{dt}$ $\Rightarrow \frac{dr}{r} = \frac{3}{4}d\theta$

Integrating $\int\frac{dr}{r} = \frac{3}{4}d\theta$

$r = e^{\frac{3}{4}\theta}$

where c is arbitrary constant which can be evaluated by using initial condition

$r = 1$ at $\theta = 0 \Rightarrow c = 1$

So, $r = e^{\frac{3}{4}\theta}$ Equation of trajectory $r(\theta)$

Magnitude of velocity $|\vec{v}| = 5t$

$\sqrt{v_r^2 + v_\theta^2} = 5t$

$\Rightarrow \sqrt{\frac{9}{16}v_\theta^2 + v_\theta^2} = 5t$ $\left(\text{Since } v_r = \frac{3}{4}v_\theta\right)$

$\Rightarrow \frac{5}{4}v_\theta = 5t$ $\Rightarrow v_\theta = 4t$ $\Rightarrow r\frac{d\theta}{dt} = 4t$ $\Rightarrow e^{\frac{3}{4}\theta}d\theta = 4t\,dt$

Integrating

$\Rightarrow \int e^{\frac{3}{4}\theta}d\theta = t\,dt$ $\Rightarrow \frac{4}{3}e^{\frac{3}{4}\theta} = 2t^2 + c$

where c is arbitrary constant which can be evaluated using initial condition

$\theta = 0$ at $t = 0$ $\Rightarrow c\frac{4}{3}$

So, $e^{\frac{3}{4}\theta} = \frac{3}{2}t^2 + 1$

At θ = 4 radian

$e^3 = \frac{3}{2}t^2 + 1 \qquad \Rightarrow \quad t = \sqrt{\frac{38}{3}}\,\text{sec}.$

So, θ = 4 radian at t = 3.55 sec

48. - 3.75

Central Force potential energy

$$V(r) = -\frac{\exp\left(-\frac{3r}{2}\right)}{5r^3}$$

Effective potential = V(r) + $\frac{L^2}{2mr^2}$

$$V_e = -\frac{\exp\left(-\frac{3r}{2}\right)}{5r^2} + \frac{L^2}{2mr^2} = -\frac{e^{-\frac{3r}{2}}}{5r^2} + \frac{L^2}{2r^2} \qquad (m = 1\text{ kg})$$

The body moves in a circular orbit of radius r = 2m

So, energy of the system will always be equal to effective potential V_e at r =2m

Also, V_e will be minimum at r = 2m and $\frac{dV_e}{dr}$ will be zero at r = 2m

$$\frac{dV_e}{dr} = \frac{2e^{-\frac{3r}{2}}}{5r^3} + \frac{3}{10}.\frac{e^{-\frac{3r}{2}}}{r^2} - \frac{L^2}{r^3}$$

$\frac{dV_e}{dr}$ at r = 2m will be zero

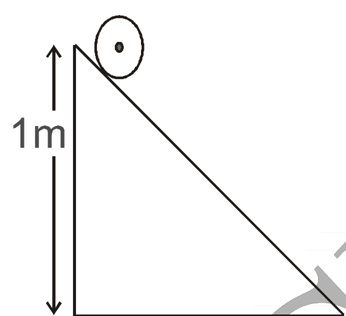
$$\Rightarrow \frac{2e^{-\frac{3r}{2}}}{5r^3} + \frac{3}{10}.\frac{e^{-\frac{3r}{2}}}{r^2} - \frac{L^2}{r^3} = 0 \qquad \Rightarrow \frac{2e^{-3}}{40} + \frac{3}{10} x \frac{e^{-3}}{4} - \frac{L^2}{8}$$

$$\Rightarrow L^2 = e^{-3} = \frac{1}{20} \qquad \Rightarrow L = \frac{1}{\sqrt{20}} kg.m^2 s^{-1}$$

For circular motion

$$E = V_e \text{ at } r = 2m = -\frac{e^{-3}}{20} + \frac{1}{20x2x4} = -\frac{1}{400} + \frac{1}{160} = -\frac{3}{800} J = -3.75 mJ$$

49. 3.6



The kinetic energy of cylinder when it reaches the bottom = potential energy of the cylinder at the top = mgh = 1 x 9.8 x 1 = 9.8 J

Kinetic energy kE = kE of translation + kE of rotation

$$= \frac{1}{2} mV_{cm}^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} mV_{cm}^2 + \frac{1}{2}.\frac{1}{2} mR^2.\frac{V_{cm}^2}{R^2}$$

$$= \frac{1}{2} mV_{cm}^2 + \frac{1}{4} mV_{cm}^2 = \frac{3}{4} mV_{cm}^2$$

Kinetic energy at bottom = 9.8 J

$$\Rightarrow \frac{3}{4} mV_{cm}^2 = 9.8$$

$$V_{cm} = \sqrt{\frac{9.8x4}{3}} = 3.6 m/s$$

50. 772.55

Wavelength of X-ray, λ = 0.612 x 10^{-10} m = 0.612 A^0

Compton shift in wavelength

$$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\phi)$$

h = planck's constant

m_0 = rest mass of electron

c = speed of light

ϕ = Angle b which radiation gets scattered

So, $\Delta\lambda = \frac{h}{m_0 c}(1-\cos 90^o)$ = 2.426 x 10^{-12} m = 0.02426 A^o

Kinetic energy imparted to the electron

KE = Change in energy possessed by photon

$$= \frac{hc}{\lambda} - \frac{hc}{\lambda'} = hc\frac{1}{\lambda} - \frac{1}{\lambda'} = 124 \text{ eV - nm}\left[\frac{1}{0.612} - \frac{1}{0.63626}\right] = 772.55 \text{ eV}$$

51. 2

Resolving power required to resolve two wavelengths = $\frac{\lambda}{d\lambda} = \frac{5000}{0.5} - 10000$

So, Resolving power of grating must be greater than 10000

Resolving power of grating

$$R = Nn \geq 10000$$

$$n \geq \frac{10000}{5000x1.8} = \frac{10}{9} = 1.11$$

So, least order at which two wavelengths are resolved n_{min} = 2

52. 80

$$\text{CMRR} = 20 \log \frac{|Ad|}{|Ac|} = 20\log\frac{3500}{0.35} = 20\log 10^4$$

CMRR = 80dB

53. 3

$$V_0 = -R_f\left[\frac{V_a}{R_1}+\frac{V_b}{R_2}\right] = -10k\left[-\frac{4}{10k}-\frac{2}{20k}\right]$$

$[V_0 = +3V]$

54. 150

The mass of an aircraft = 12000 kg

Uniform speed of aircraft is $540\times\frac{5}{18} = 150\,m/sec$

The distance between the last two packet drops will be equal to the covered distance in one second = 150 m.

55. 519

Given $C_P = \left(1+\frac{T}{5}\right)\frac{J.deg}{C\,mol}$

$m_1 = 0.2$, $T_1 = 600$ C
$m_2 = 0.1$, $T_2 = 300$ C
We know at the thermal equilibrium
Heat absorb = Heat gain

$$\int_{600}^{T}(0.2)\left(1+\frac{T}{5}\right)dT = -\int_{300}^{T}(0.1)\left(1+\frac{T}{5}\right)dT$$

$$2\left[T+\frac{T^2}{10}\right]_{600}^{T} = -\left[T+\frac{T^2}{10}\right]_{300}^{T}$$

$$2T+T+\frac{2T^2}{10}+\frac{T^2}{10} = 300+\frac{(300)^2}{10}+1200+\frac{2}{10}(600)^2$$

$$\frac{3T^2}{10}+3T = 1500+\frac{1}{10}\left[300^2+2(600)^2\right]$$

$3T^2 + 30T = 15000 + [300^2 + 2(600)^2]$
$T = 519$

56. 84.70

We know that $g' = g - \omega^2 R\cos^2\lambda$

At equator $g' = g - \omega^2 R$

If $\omega' = 17\omega$ then the object become weightloss at the equator .

Then the time period of rotation of this planet in minutes is

$$T = \frac{2\pi}{\omega} = \frac{24}{17} = 84.70 \text{ min}$$

57. 24.2

Since, V_E = 5.8 V always remains constant, therefore maximum load current when volts (V) is maximum i.e. = 30V

$\therefore$ 30 = 1000 x I_{max} + 5.8

$\therefore$ I_{max} = 24.2 mA

58. 395.3

PIV across each diode = 2 x 280 $\sqrt{2}$ Volts.

when the diode is conducting (during half cycle) the voltage across the diode is zero.

Hence, rms value of the voltage of the voltage across the diode

$$= \frac{PIV}{2} = 280\sqrt{2} = 395.3V$$

59. 5.2

$$I_C = \frac{I_C}{1-\alpha} + \frac{\alpha}{1-\alpha} I_B = \frac{6}{1-0.98} + \frac{0.98}{1-0.98} \times 100 \quad = 5.2\text{mA}$$

60. - 7

$R_f = 10k\Omega$

and R = R_1 = R_2 = 1.0 $k\Omega$

Therefore, $V_{out} = -\frac{R_f}{R}(V_{IN\,(A)} + V_{IN\,(B)}) = -\frac{10k\Omega}{1.0k\Omega}(0.2V + 0.5V)$

$= -10\,(0.7V) = -7V$